MDS
SR. SEC. SCHOOL

SAMPLE PAPER
PHYSICS

For Annual Exam
Class - XI

Multiple choice questions

1. A constant retarding force 100 N is applied to a body of mass 20 kg, moving initially with speed 20 m/s. How long does the body take to stop?
(1) 2 s (2) 3 s (3) 1 s (4*) 4 s

2. A force F = 20 N acts on a object and displaces it from rest to speed of 10 m/s in its direction. What is displacement, if mass of object is 2 kg?
(1) 6 m (2*) 5 m (3) 12 m (4) 10 m

3. A bob of mass m is suspended by light string of length l. At lowest position it is imparted a horizontal velocity $\sqrt{5gl}$ such that it just completes circular trajectory in vertical circle. What is ratio of its KE at B and C?
(1) 2 : 1 (2*) 3 : 1 (3) 5 : 3 (4) 3 : 2

4. Three particles of equal masses are placed at co-ordinates (1, 1), (2, 2) and (4, 4) respectively. The position co-ordinate of COM of system of three particles is
(1) (0, 0) (2) $\left(\frac{2}{7}, \frac{7}{2}\right)$ (3*) $\left(\frac{7}{3}, \frac{7}{3}\right)$ (4) (2, 2)

5. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration a. The centre of mass has an acceleration
(1) Zero (2*) $\frac{1}{2}a$ (3) a (4) 2 a

6. Hooke's law is applicable for
(1) Plastic materials (2*) Elastic materials
(3) Both (a) and (b) (4) Brittle materials

7. The maximum displacement of a particle executing S.H.M is 2 cm and the maximum acceleration is 8 cm/sec^2, then the time period is
(1*) 3.14 sec (2) 4.00 sec (3) 1.57 sec (4) 2.0 sec

8. A cord of negligible mass is wrapped around a solid cylinder of a mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on cord tangentially. The cylinder is mounted on horizontal axis with frictionless bearings. What is kinetic energy of wheel when 2 m cord is unwound?
(1*) 50 J (2) 100 J (3) 150 J (4) 90 J

9. Four bodies; a ring, a solid cylinder, a hollow sphere and a solid sphere of same mass are allowed to roll down a rough inclined plane without slipping from same level. The body with greatest rotational kinetic energy at bottom is
(1*) Ring (2) Solid cylinder (3) Hollow sphere (4) Solid sphere

10. A ring (circular) of radius 2 m has mass of 100 kg. It rolls purely along horizontal floor so that its COM has speed 20 cm s^{-1}. The work required to stop it is
(1) 2 J (2) 3 J (3*) 4 J (4) 8 J

11. Which of the following materials is/are close to ideal plastics?
(1) Putty (2) Mud (3) Steel (4*) Both (1) & (2)

12. The ratio of stress and strain, within proportional limit is called
(1*) Modulus of elasticity (2) Compressibility
(3) Poisson's ratio (4) Both (2) & (3)

13. A and B are two wires. The radius of A is thrice that of B. they are stretched by the same load. Then the stress on B is

(1) Equal to that on A
(2*) Nine times that on A
(3) Two times that on A
(4) Half that on A

14. Along a streamline

(1) the velocity of a fluid particle remains constant.
(2*) the velocity of all fluid particles crossing a given position is constant.
(3) the velocity of all fluid particles at a given instant is constant.
(4) the speed of a fluid particle remains constant

15. The equation of motion is represented by y = sin ωt + cos ωt. The time period of periodic motion is

(1) $\frac{\pi}{\omega}$
(2*) $\frac{2\pi}{\omega}$
(3) 2π
(4) $\frac{4\pi}{\omega}$

16. A particle executes SHM. Its time period is T. The kinetic energy of the particle is also periodic with time period of

(1) T
(2) 2T
(3*) $\frac{T}{2}$
(4) Infinity

17. Force exerted on a body can changes it's

(a) Direction of motion
(b) Momentum
(c) Kinetic energy
(d*) All the above

18. A truck and car are moving on a plane road with same kinetic Energy. They are brought to rest by application of brakes which provide equal retarding forces. Which of the following statements is true

(a) Distance traveled by truck is shorter then car before coming to rest
(b) Distance traveled by car is shorter then truck before coming to rest
(c) Distance traveled depends on individual velocity of both the vehicles
(d*) Both will travel same distance before coming to rest

19. A body of mass 10 kg is initially at a height of 2 m above the ground. It is lifted to a height of 2 m from that position. Its increase in potential energy is (g = 9.8 m/s^2):

(a) 100 J
(b*) 196 J
(c) 60 J
(d) -100 J

20. A wheel is at rest. Its angular velocity increases uniformly and becomes 50 rad/sec after 5 sec. The total angular displacement is

(a) 600 rad
(b) 75 rad
(c) 300 rad
(d*) 125 rad

21. Ten particles of mass = 1 kg each are attached to the rim of a circular disc of radius 0.1 m and negligible mass. Moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to its plane is

(a) 1 kg m^2
(b*) 0.1 kg m^2
(c) 2 kg m^2
(d) 0.2 kg m^2

22. A wire is stretched by x when a load F is hanged on it. If the same wire goes over a pulley and two weight F each are hung at the two end's the what will be the elongation in the wire

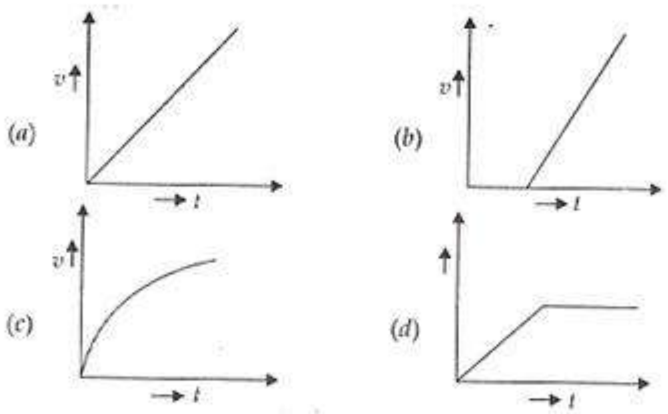
(a*) x
(b) 2x
(c) x/2
(d) 0

23. A block A of mass m_1 is released from top of smooth inclined plane and it slides down the plane. Another block of mass m_2 such that $m_2 > m_1$ is dropped from the same point and falls vertically downwards. Which one of the following statements will be true if the friction offered by air is negligible?

(a) Both blocks will reach ground at same time
(b*) Both blocks will reach ground with the same speed

(c) speed of both the blocks when they reach ground will depend on their masses
(d) Block A reaches ground before block B

24. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in Figure, indicate the one that represents the velocity v of the pebble as a function of time (t).



Ans. **(c)**

ASSERTION & REASON QUESTIONS

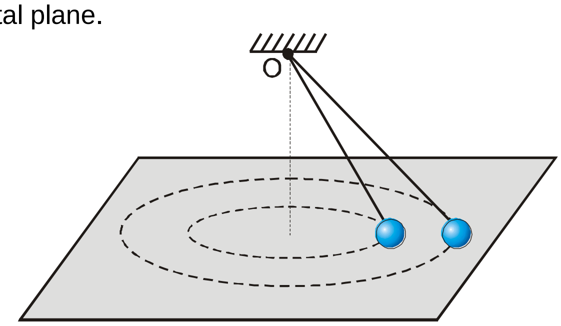
Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:
(a)Both A and R are true and R is the correct explanation of A
(b)Both A and R are true but R is not the correct explanation of A
(c)A is true but R is false
(d)A is false and R is also false

25. **Assertion :** The centre of mass of a body may lie where there is no mass.
Reason : Centre of mass of body is a point, where the whole mass of the body is supposed to be concentrated.
(a*) (b) (c) (d)

26. **Assertion :** Bulk modulus of elasticity (B) represents incompressibility of the material.
Reason : Bulk modulus of elasticity is proportional to change in pressure.
(a*) (b) (c) (d)

27. **Assertion :** When a particle is at extreme position performing SHM, its momentum is equal to zero.
Reason : At extreme position the velocity of particle performing SHM is equal to zero
(a*) (b) (c) (d)

28. **Assertion :** Two small spheres are suspended from same point O on roof with strings of different lengths. Both spheres move along horizontal circles as shown. Then both spheres may move along circles in same horizontal plane.



Reason : For both spheres in statement-1 to move in circular paths in same horizontal plane, their angular speeds must be same.

(a*) (b) (c) (d)

29. **Assertion :** The centre of mass of a proton and an electron released from their respective positions remains at rest.

Reason : The centre of mass remain at rest, if no external force is applied.

(a*) (b) (c) (d)

Very short Answer type question

30.

A body is being raised to a height h from the surface of earth. What is the sign of work done by

(a) applied force and

(b) gravitational force?

Ans. **(a)** Force is applied on the body to lift it in upward direction and displacement of the body is also in upward direction, therefore, angle between the applied force and displacement is $\theta = 0°$

$\therefore$ Work done by the applied force

$$W = Fs\cos\theta = Fs\cos 0° = Fs \qquad (\because \cos 0° = 1)$$

i.e., W = Positive

(b) The gravitational force acts in downward direction and displacement in upward direction, therefore, angle between them is $\theta = 180°$.

$\therefore$ Work done by the gravitational force

$$W = Fs\cos 180° = -Fs \qquad (\because \cos 180° = 1)$$

31.

Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

Ans. Force of gravity acts on the car vertically downward while car is moving along horizontal road, *i.e.,* angle between them is 90°.

Work done by the car against gravity

$$W = Fs\cos 90° = 0 \qquad (\because \cos 90° = 0)$$

32.

Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s.

Ans. Given,

$$\text{mass} = m = 100\,\text{kg}$$
$$\text{height} = h = 10\,\text{m} \quad \text{time duration } t = 20\text{s}$$
$$\text{power} = \text{Rate of work done}$$
$$= \frac{\text{change of PE}}{\text{time}} = \frac{mgh}{t}$$
$$= \frac{100 \times 9.8 \times 10}{20}$$
$$= 5 \times 98 = 490\ \text{W}$$

33. Is viscosity a vector?

Ans. Viscosity is a property of liquid it does not have any direction, hence it is a scalar quantity.

34. **(a)** A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?
(b) Find impulse of force in this duration.

Ans. **(a)**

Here, $m = 3.0$ kg, $u = 2.0\ \text{ms}^{-1}$
$v = 3.5\ \text{ms}^{-1}$, $t = 25$ s

As $F = ma$

or $F = m\left(\frac{v-u}{t}\right)$ $\left[\because a = \frac{v-u}{t}\right]$

$\Rightarrow$ $F = \frac{3.0\,(3.5-2.0)}{25} = 0.18\,\text{N}.$

The force is along the direction of motion.

(b) 4.5 Ns

35. A body is moving along Z – axis of a co – ordinate system is subjected to a constant force F is given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N Where I, j ,k are unit vector along the x, y and z axis of the system respectively what is the work done by this force in moving the body a distance of 4m along the Z axis?

Ans: $\overline{F} = -\hat{i} + 2\hat{j} + 3\hat{k}\ N,$

$\hat{S} = 4\hat{k}$

$W = \overline{F}.\overline{S}$

$W = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right).\left(4\hat{k}\right)$

W = 12 J

36. **(a)** What do you mean by momemt of force? Define a couple?
(b) What is point of application of force?

Ans. **(a)** A couple is a pair of forces, equal in magnitude, whose line of action of force is not the same. Moment of couple is equal to the product of either of forces and the perpendicular distance between the forces. Its SI unit is Nm

37. A system consisting of two objects has a same mass if velocity of objects are $10\hat{i}\ m/s$ and $10\hat{j}\ m/s$, then find out speed of center of mass.

Ans. $\vec{V}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$= \dfrac{10\hat{i} + 10\hat{j}}{2} = 5\hat{i} + 5\hat{j}$

so speed of center of mass = $5\sqrt{2}\,m/s$

38. In case of an oscillating simple pendulum what will be the direction of acceleration of the bob at (a) the mean position, (b) the end points?

Ans. a. The direction of acceleration of the bob at its mean position is completely radial i.e. towards the point of suspension along the thread or string. Whereas the transverse acceleration is zero over there.
b. At extreme points, however, the acceleration is tangential towards the mean position, which helps to bring the bob back to the equilibrium position.

39. Formula of instantaneous power in terms of force and velocity.

Ans. $P = \vec{F}.\vec{v}$

Short Answer type question

40.

Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

Ans. The moment of inertia of a body is given by $I = \Sigma m_i r_i^2$ [sum of moment of inertia of each constituent particles]
All the mass in a cylinder lies at distance R from the axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R.

41.

The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?

Ans. Young's modulus $(Y) = \dfrac{\text{Stress}}{\text{Longitudinal strain}}$

For same longitudinal strain, $Y \propto \text{stress}$

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} = \frac{(\text{stress})_{\text{steel}}}{(\text{stress})_{\text{rubber}}}$$

But $Y_{\text{steel}} > Y_{\text{rubber}}$

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} > 1$$

Therefore, from Eq. (i),

$$\frac{(\text{stress})_{\text{steel}}}{(\text{stress})_{\text{rubber}}} > 1$$

$$\Rightarrow \quad (\text{stress})_{\text{steel}} > (\text{stress})_{\text{rubber}}$$

42.

Identical springs of steel and copper are equally stretched. On which, more work will have to be done?

Ans. Work done in stretching a wire is given by $W = \frac{1}{2} F \times \Delta l$

[where F is applicd force and Δl is extension in the wire]

As springs of steel and copper are equally stretched. Therefore, for same force (F),

$$W \propto \Delta l \qquad \ldots\text{(i)}$$

$$\text{Young's modulus } (Y) = \frac{F}{A} \times \frac{l}{\Delta l} \Rightarrow \Delta l = \frac{F}{A} \times \frac{l}{Y}$$

As both springs are identical, $\Delta l \propto \dfrac{1}{Y}$ ...(ii)

From Eqs. (i) and (ii), we get $W \propto \dfrac{1}{Y}$

$$\therefore \quad \frac{W_{\text{steel}}}{W_{\text{copper}}} = \frac{Y_{\text{copper}}}{Y_{\text{steel}}} < 1 \qquad (\text{As, } Y_{\text{steel}} > Y_{\text{copper}})$$

$$\Rightarrow \quad W_{\text{steel}} < W_{\text{copper}}$$

Therefore, more work will be done for stretching copper spring.

43.

Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged, if the density of ice is $\rho_i = 0.917 \text{ g cm}^{-3}$?

Ans. Given, density of ice (ρ_{ice}) = 0.917g /cm^3

Density of water (ρ_w) = 1 g/cm^3

Let V be the total volume of the iceberg and V' of its volume be submerged in water.

In floating condition.

Weight of the iceberg = Weight of the water displaced by the submerged part by ice

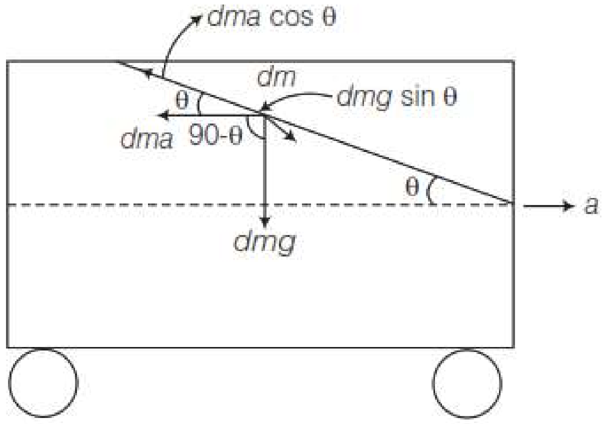
$$V\rho_{ice}g = V'\rho_w g$$

or $$\frac{V'}{V} = \frac{\rho_{ice}}{\rho_w} = \frac{0.917}{1} = 0.917 \qquad (\because \text{ Weight} = mg = v\rho g)$$

44.

The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be titled by an angle θ. If the acceleration is a ms^{-2}, what will be the slope of the free surface?

Ans. Consider the diagram where a tanker is accelerating with acceleration a.



Consider an elementary particle of the fluid of mass dm.

The acting forces on the particle with respect to the tanker are shown above .

Now, balancing forces (as the particle is in equilibrium) along the inclined direction component of weight = component of pseudo force $dmg\sin\theta = dma\cos\theta$ (we have assumed that the surface is inclined at an angle θ) where, dma is pseudo force

$\Rightarrow$ $$g\sin\theta = a\cos\theta$$

$\Rightarrow$ $$a = g\tan\theta$$

$\Rightarrow$ $$\tan\theta = \frac{a}{g} = \text{slope}$$

45. State and prove work energy theorem analytically?

Ans.

Ans: It states that work done by force acting on a body is equal to the change produced in its kinetic energy.

If $\overline{F}$ force is applied to move an object through a distance dS

Then $dw = \overline{F}.\overline{dS}$

$F = m\bar{a}$

$dw = m\bar{a}.d\overline{S}$

$dw = m\frac{d\bar{v}}{dt}.d\overline{S}$

$dw = m\frac{ds}{dt}d\upsilon$

$dw = m\upsilon d\upsilon$

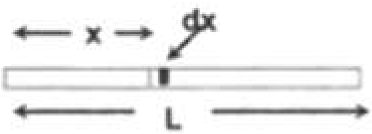
Integrating

$\int_0^W dW = W = \int_u^v m\upsilon d\upsilon$

$W = m\left|\frac{V^2}{2}\right|_u^v$

46. If the linear mass density of a rod of length L varies as $\lambda = A + Bx$ (where A and B are constants), then find the coordinates of center of mass.

Ans.



To calculate center of mass for a continuous system of particles let us consider infinitesimal small element of length dx and mass dm and then use the center of mass equation below to calculate the center of mass for X-Coordinate.

$$X_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm}$$

$$dm = \lambda dx = (A + Bx)dx$$

$$X_{CM} = \frac{\int_0^L x(A + Bx)dx}{\int_0^L (A + Bx)dx} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}}$$

$$X_{CM} = \frac{L(3A + 2BL)}{3(2A + BL)}$$

47. (a) What is longitudinal stress?
(b) Define Bulk modulus.

Ans. (a) longitudinal stress = $\frac{\text{force}}{\text{cross section area}}$

force is perpendicular to surface

(b) bulk modulus = $\frac{\text{change in pressure}}{\text{volumetric strain}}$

$$B = \frac{\Delta P}{\frac{\Delta v}{v}}$$

unit of bulk modulus is $\frac{\text{Newton}}{\text{meter}^2}$

48. Find out pressure at 20 meter depth of a lake. If atmospheric pressure is 10^5 Pa and density of water is 10^3 kg/m^3 (g = 10 m/s^2)

Ans. Let pressure at 10 meter depth is P

$P = P_0 + \rho gh$

$P = 10^5 + (10^3 \times 20 \times 10)$

$P = 3 \times 10^5$ Pa

49. **(a)** The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5s? (g on the surface of earth is 9.8 ms^{-2})
(b) If on any other planet acceleration due to gravity is 6.8 m/s^2 then time period of simple pendulum on that planet will be-

Ans. **(a)**

12. Acceleration due to gravity on the surface of moon, g' = 1.7 ms^{-2}

Acceleration due to gravity on the surface of earth, g = 9.8 m s^{-2}

Time period of a simple pendulum on earth, T = 3.5 s

Time period of simple pendulum is given as:

$$\Rightarrow T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{\frac{m}{mg}}{l}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

⇒l is the length of the pendulum

$$\Rightarrow l = \frac{T^2}{(2\pi)^2} \times g$$

$$= \frac{(3.5)^2}{4\times(3.14)^2} \times 9.8m$$

Since ,The length of the pendulum remains constant.

On moon's surface, time period will be

$$\Rightarrow T' = 2\pi\sqrt{\frac{l}{g'}}$$

$$= 2\pi\sqrt{\frac{(3.5)^2}{4\times(3.14)^2}} \times 9.8m = 8.4s$$

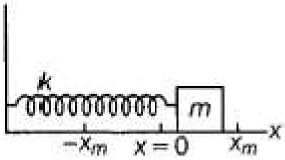
(b) 7 sec.

50. Consider a block of mass 700 g is fastened to a spring having spring constant of 70 N/m. Find out the following parameters if block is pulled a distance of 14 cm from its mean position on a frictionless surface and released from rest at t =0. 5

i. The angular frequency, the frequency and the period of the resulting motion.

ii. The amplitude of the oscillation.

iii. The maximum speed of the oscillating block.



Ans.

i. The angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{70\text{N/m}}{0.700\text{kg}}} = 10 \text{ rad/s}$$

Frequency, $f = \frac{\omega}{2\pi} = \frac{10}{2\pi} \simeq 1.59\text{Hz}$

The time period, $T = \frac{1}{f} = \frac{1}{1.59} = 0.63 = 630$ ms

ii. The maximum amplitude of the oscillation = maximum displacement

$\therefore x_m = 14$ cm $= 0.14$ m

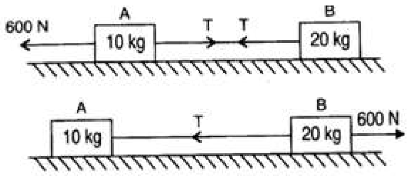
iii. The maximum speed of the oscillation v_m is given by

$v_m = \omega x_m = 10 \times 0.14 = 1.4$ m/s

51. Draw stress strain curve and indicate yield point and fracture point of a material.

52. State Pascal's law and explain working of hydraulic lift.

53. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a tight string. A horizontal force F = 600 N is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?



Ans.

$$\text{Acceleration} = \frac{600\text{ N}}{10\text{ kg} + 20\text{ kg}} = 20\text{ ms}^{-2}$$

(*i*) When force is applied on 10 kg mass

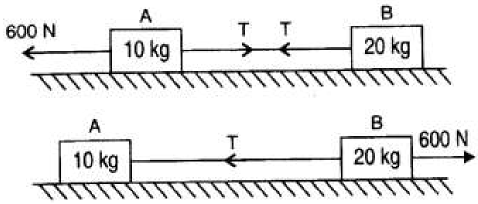
$600 - T = 10 \times 20$ or

$T = 400$ N

(*ii*) When force is applied on 20 kg mass,

$600 - T = 20 \times 20$ or

$T = 200$ N



Long answer type question

54.

What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?

Ans. Let $x = A\sin\omega t$ is the displacement function of SHM.

Velocity, $$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$v_{max} = A\omega\,|\cos\omega t|_{max}$$

$$= A\omega \times 1 = \omega A \qquad [\because |\cos\omega t|_{max} = 1] \quad \ldots(i)$$

Acceleration, $a = \frac{dv}{dt} = -\omega A \cdot \omega\sin\omega t$

$$= -\omega^2 A\sin\omega t$$

$$|a_{max}| = |(-\omega^2 A)(+1)| \qquad [\because (\sin\omega t)_{max} = 1]$$

$$|a_{max}| = \omega^2 A \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{v_{max}}{a_{max}} = \frac{\omega A}{\omega^2 A} = \frac{1}{\omega}$$

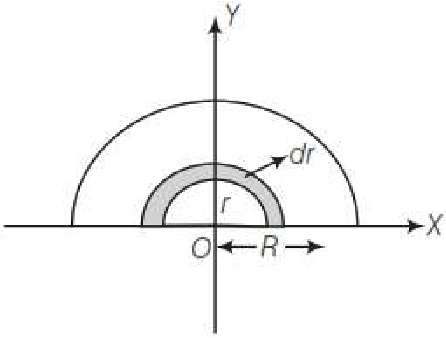
$$\Rightarrow \qquad \frac{a_{max}}{v_{max}} = \omega$$

55.

Find the centre of mass of a uniform (a) half-disc,

Ans. Let M and R be the mass and radius of the half-disc, mass per unit area of the half-disc

$$m = \frac{M}{\frac{1}{2}\pi R^2} = \frac{2M}{\pi R^2}$$



(a) The half-disc can be supposed to be consists of a large number of semicircular rings of mass dm and thickness dr and radii ranging from $r = 0$ to $r = R$.

Surface area of semicircular ring of radius r and of thickness $dr = \frac{1}{2} 2\pi r \times dr = \pi r dr$

$\therefore$ Mass of this elementary ring, $dm = \pi r dr \times \frac{2M}{\pi R^2}$

$$dm = \frac{2M}{R^2} r dr$$

If (x, y) are coordinates of centre of mass of this element,

then, $$(x, y) = \left(0, \frac{2r}{\pi}\right)$$

Therefore, $$x = 0 \text{ and } y = \frac{2r}{\pi}$$

Let x_{CM} and y_{CM} be the coordinates of the centre of mass of the semicircular disc.

Then $$x_{CM} = \frac{1}{M}\int_0^R x\,dm = \frac{1}{M}\int_0^R 0\,dm = 0$$

$$y_{CM} = \frac{1}{M}\int_0^R y\,dm = \frac{1}{M}\int_0^R \frac{2r}{\pi} \times \left(\frac{2M}{R^2}\,r\,dr\right)$$

$$= \frac{4}{\pi R^2}\int_0^R r^2 dr = \frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{4}{\pi R^2} \times \left(\frac{R^3}{3} - 0\right) = \frac{4R}{3\pi}$$

$\therefore$ Centre of mass of the semicircular disc $= \left(0, \frac{4R}{3\pi}\right)$

56.

A body of mass m is situated in a potential field $U(x) = U_0\ (1 - \cos\alpha x)$ when, U_0 and α are constants. Find the time period of small oscillations.

Ans. Given potential energy associated with the field

$$U(x) = U_0\ (1 - \cos\alpha x) \quad \text{..... (i)}$$

Now, force $F = -\dfrac{dU(x)}{dx}$

[$\because$ for conservatine force f, we can write $f = \dfrac{-du}{dx}$]

[We have assumed the field to be conservative]

$$F = -\frac{d}{dx}(U_0 - U_0 \cos\alpha x) = -U_0\ \alpha \sin\alpha x$$

$$F = -U_0\ \alpha^2 x \quad \text{... (ii)}$$

[$\because$ for small oscillations αx is small, $\sin\alpha x \approx \alpha x$]

$\Rightarrow$ $F \propto (-x)$

As, U_0, α being constant.

$\therefore$ Motion is SHM for small oscillations.

Standard equation for SHM $F = -m\omega^2 x$ ...(iii)

Comparing Eqs. (ii) and (iii), we get

$$m\omega^2 = U_0\alpha^2$$

$$\omega^2 = \frac{U_0\alpha^2}{m} \text{ or } \omega = \sqrt{\frac{U_0\alpha^2}{m}}$$

$\therefore$ Time period $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{U_0\alpha^2}}$

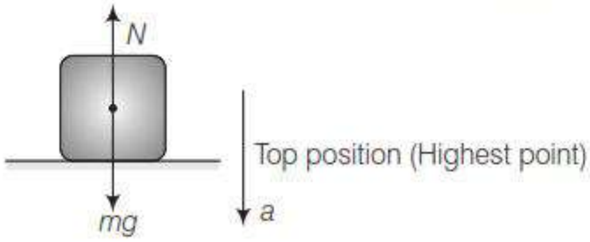
57.

A person normally weighing 50 kg stands on a massless platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. A weighing machine on the platform gives the persons weight against time.

(a) Will there be any change in weight of the body, during the oscillation?

(b) If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?

Ans. In this case acceleration is variable. In accelerated motion, weight of body depends on the magnitude and direction of acceleration for upward or downward motion.



(a) Hence, the weight of the body changes during oscillations

(b) Considering the situation in two extreme positions, as their acceleration is maximum in magnitude.

we have, $mg - N = ma$

$\because$ At the highest point, the platform is accelerating downward.

$\Rightarrow$ $N = mg - ma$

But $a = \omega^2 A$ [in magnitude]

$\therefore$ $N = mg - m\omega^2 A$

where, A = amplitude of motion.

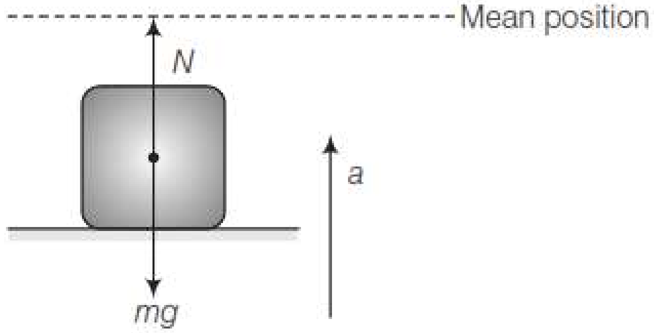
Given, $m = 50$ kg, frequency $\nu = 2\ s^{-1}$

$\therefore$ $\omega = 2\pi\nu = 4\pi$ rad/s

$A = 5\text{ cm} = 5 \times 10^{-2}$ m

$\therefore$
$$N = 50 \times 9.8 - 50 \times (4\pi)^2 \times 5 \times 10^{-2}$$
$$= 50[9.8 - 16\pi^2 \times 5 \times 10^{-2}]$$
$$= 50[9.8 - 7.89]$$
$$= 50 \times 1.91$$
$$= 95.5\text{ N}$$

When the platform is at the lowest position of its oscillation,



It is accelerating towards mean position that is vertically upwards.

Writing equation of motion

$$N - mg = ma = m\omega^2 A$$

or $\quad N = mg + m\omega^2 A$

$$= m[g + \omega^2 A]$$

Putting the data $\quad N = 50[9.8 + (4\pi)^2 \times 5 \times 10^{-2}]$

$$= 50[9.8 + (12.56)^2 \times 5 \times 10^{-2}]$$

$$= 50[9.8 + 7.88]$$

$$= 50 \times 17.68 = 884 \text{ N}$$

Now, the machine reads the normal reaction. It is clear that

maximum weight = 884 N (at lowest point)

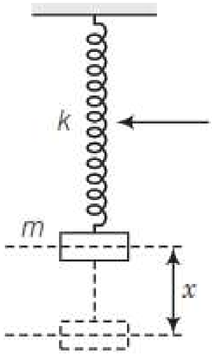
minimum weight = 95.5 N (at top point)

58.

A body of mass *m* is attached to one end of a massless spring which is suspended vertically from a fixed point. The mass is held in hand, so that the spring is neither stretched nor compressed. Suddenly, the support of the hand is removed. The lowest position attained by the mass during oscillation is 4 cm below the point, where it was held in hand.

(a) What is the amplitude of oscillation?

Ans. (a) When the support of the hand is removed, the body oscillates about a mean position.



Suppose x is the maximum extension in the spring when it reaches the lowest point in oscillation.

Loss in PE of the block $= mgx$ ...(i)

where, $m =$ mass of the block

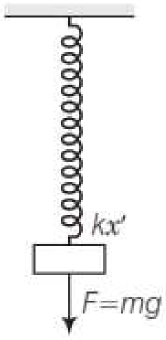
Gain in elastic potential energy of the spring

$$= \frac{1}{2}kx^2 \quad \text{...(ii)}$$

As the two are equal, conserving the mechanical energy,

we get, $$mgx = \frac{1}{2}kx^2 \text{ or } x = \frac{2mg}{k} \quad \text{...(iii)}$$

Now, the mean position of oscillation will be, when the block is balanced by the spring.



If x' is the extension in that case, then

$$F = +kx'$$

But $$F = mg$$

$\Rightarrow$ $$mg = +kx'$$

or $$x' = \frac{mg}{k} \quad \text{...(iv)}$$

Dividing Eq. (iii) by Eq. (iv),

$$\frac{x}{x'} = \frac{2mg}{k} / \frac{mg}{k} = 2$$

$\Rightarrow$ $$x = 2x'$$

But given $x = 4$ cm (maximum extension from the unstretched position)

$\therefore$ $$2x' = 4$$

$\therefore$ $$x' = \frac{4}{2} = 2 \text{ cm}$$

But the displacement of mass from the mean position to the position when spring attains its natural length is equal to amplitude of the oscillation.

$\therefore$ $$A = x' = 2 \text{ cm}$$

where, A = amplitude of the motion.

59.

A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 m s^{-1}. Calculate

(a) the loss of PE of the drop.

(b) the gain in KE of the drop.

(c) Is the gain in KE equal to loss of PE? If not why?

Take, $g = 10 \ ms^{-2}$.

Ans. Given, mass of the rain drop $(m) = 1.00$ g

$$= 1 \times 10^{-3} \text{ kg}$$

Height of falling $(h) = 1 \text{ km} = 10^3$ m

$$g = 10 \text{ m/s}^2$$

Speed of the rain drop $(v) = 50$ m/s

(a) Loss of PE of the drop $= mgh$

$$= 1 \times 10^{-3} \times 10 \times 10^3 = 10\text{J}$$

(b) Gain in KE of the drop $= \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 1 \times 10^{-3} \times (50)^2$$

$$= \frac{1}{2} \times 10^{-3} \times 2500$$

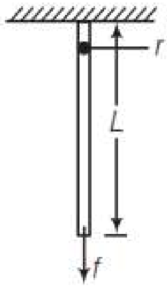
$$= 1.250 \text{ J}$$

(c) *No*, gain in KE is not equal to the loss in its PE, because a part of PE is utilised in doing work against the viscous drag of air.

60.

A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

Ans. The situation is shown in the diagram.



Now, Young's modulus $(Y) = \frac{f}{A} \times \frac{L}{l}$

For first wire, $Y = \frac{f}{\pi r^2} \times \frac{L}{l}$ ...(i)

For second wire, $Y = \frac{2f}{\pi(2r)^2} \times \frac{2L}{l'}$

$= \frac{f}{\pi r^2} \times \frac{L}{l'}$ ...(ii)

From Eqs. (i) and (ii), $\frac{f}{\pi r^2} \times \frac{L}{l} = \frac{f}{\pi r^2} \times \frac{L}{l'}$

$\therefore$ $l = l'$

($\because$ Both wires are of same material, hence, Young's modulus will be same.)

61. What do you mean by banking of a curved road? Determine the angle of banking so as to minimize the wear and tear of the tyres of a car negotiating a banked curve?

Or

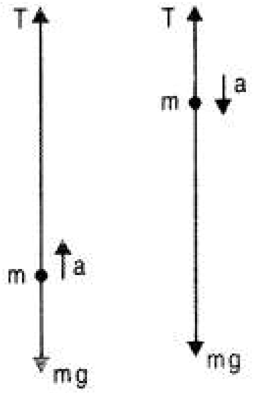
A monkey of mass 40 kg climbs on a rope (Fig.) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey
(a) climbs up with an acceleration of 6 ms^{-2}
(b) climbs down with an acceleration of 4 ms^{-2}
(c) climbs up with a uniform speed of 5 ms^{-1}
(d) falls down the rope nearly freely under gravity?
(Ignore the mass of the rope).

Ans: (a) When the monkey climbs up with an acceleration a, then T – mg = ma
where T represents the tension (figure a).

$\therefore \qquad T = mg + ma = m(g + a)$

or $\qquad T = 40 \text{ kg } (10 + 6) \text{ ms}^{-2} = 640 \text{ N}$

But the rope can withstand a maximum tension of 600 N. So the rope will break.



(*a*) (*b*)

(*b*) When the monkey is climbing down with an acceleration, then

$mg - T = ma \qquad$ (Figure (*b*))

$\Rightarrow \qquad T = mg - ma = m(g - a)$

or $\qquad T = 40 \text{ kg} \times (10 - 4) \text{ ms}^{-2} = 240 \text{ N}$

The rope will not break.

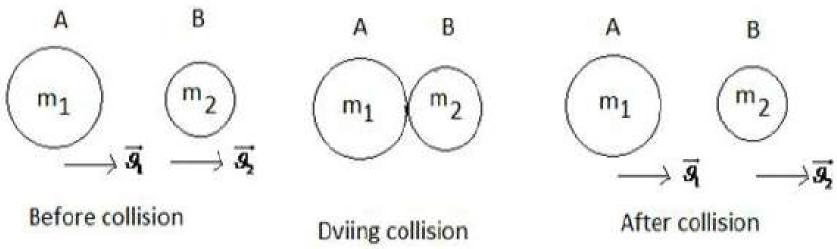
(c) When the monkey climbs up with uniform speed, then

T mg = 40 kg x 10 ms^{-2} = 400 N The rope will hot break.

(d) When the monkey is falling freely, it would be a state of weightlessness. So, tension will be zero and the rope will not break.

62. **a(i)** Prove that in an elastic collision in one dimension the relative velocity of approach before impact is equal to the relative velocity of separation after impact

Ans.



According to law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1\upsilon_1 + m_2\upsilon_2$$

$$m_1(u_1 - \upsilon_1) = m_2(\upsilon_2 - u_2) -------(1)$$

K.E. also remains conserved.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1\upsilon_1^2 + \frac{1}{2}m_2\upsilon_2^2$$

$$m_1(u_1^2 - \upsilon^2_1) = m_2(\upsilon_2^2 - u_2^2) ------(2)$$

Dividing (2) by (1)

$$u_1 - \upsilon_1 = \upsilon_2 + u_2$$

$$u_1 - u_2 = \upsilon_2 - \upsilon_1$$

I.e. Relative velocity of approach = Relative velocity of separation.

a (ii) The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that 5% of its initial energy loss against air resistance?

Ans.

Potential energy of the bob, $EP = mgl$

Kinetic energy of the bob, $EK = 0$

Total energy $= mgl$ ... (*i*)

At the lowermost point (mean position):

Potential energy of the bob, EP = 0

Kinetic energy of the bob, $E_k = \frac{1}{2}mv^2$

Total energy $E_x = \frac{1}{2}mv^2$ ... (*ii*)

As the bob moves from the horizontal position to the lowermost point, 5% of its energy gets dissipated.

The total energy at the lowermost point is equal to 95% of the total energy at the horizontal point, i.e.,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgl$$

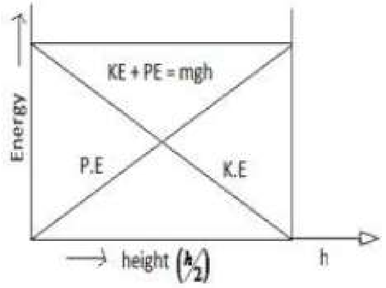
$$\therefore v = \sqrt{\frac{2 \times 95 \times 1.5 \times 9.8}{100}}$$

$$= 5.28 m/s$$

OR

b(i) Define potential energy. Give examples.
b(i) Draw a graph showing variation of potential energy, kinetic energy and the total energy of a body freely falling on earth from a height h?

Ans.



Ans: **(a)** Potential energy is the energy possessed by a body by virtue of its position in a field or due to change in its configuration example – A gas compressed in a cylinder, A wound spring of a water, water raised to the overhead tank in a house etc.

(i) Gravitational potential energy decreases as the body falls downwards and is zero at the earth

(ii) Kinetic energy increases as the body falls downwards and is maximum when the body just strikes the ground.

(iii) According to law of conservation of energy total mechanical (KE + PE) energy remains constant.

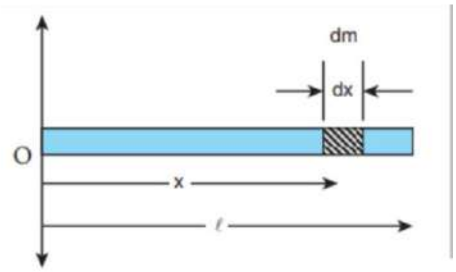
63. **(a)** Calculate by calculus for a uniform rod of mass M and length L. Its moment of inertia about an axis passing through it an end and perpendicular to length.

OR

(b) Calculate by calculus Center of mass of uniform semicircular ring of radius R.

Ans.

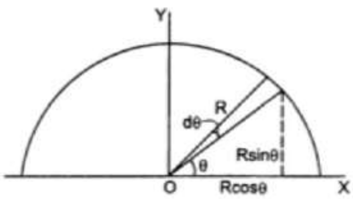
(a)



$$I = \frac{M}{\ell}\int_0^{\ell} x^2 dx = \frac{M}{\ell}\left[\frac{x^3}{3}\right]_0^{\ell} = \frac{M}{\ell}\left[\frac{\ell^3}{3}\right]$$

$$I = \frac{1}{3}M\ell^2$$

(b)



As the wire is uniform, the mass per unit length of the wire is $\frac{M}{\pi R}$. The mass of the element is, therefore,

$$dm = \left(\frac{M}{\pi R}\right)(R\,d\theta) = \frac{M}{\pi}\,d\theta.$$

The coordinates of the centre of mass are

$$X = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^{\pi}(R\cos\theta)\left(\frac{M}{\pi}\right)d\theta = 0$$

and

$$Y = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{\pi}(R\sin\theta)\left(\frac{M}{\pi}\right)d\theta = \frac{2R}{\pi}$$